La Salle Institute

Summer Math Work

For students who have completed

Calculus Honors

and are going into

AP Calculus AB

Instructions:

- You are to answer every problem.
- Your work is to be shown in the packet.
- If you need more room, attach loose leaf paper.
- Hand in completed assignment to your teacher on the first day of class.
- Your teacher will review the work.
- A review TEST will be given in the first two weeks of school, for a grade.

Thank you,
Michaela Burns
Calculus Honors Teacher

Name: _____ Date: _____

No Calculator

1. $\lim_{x\to\infty} \frac{5x^2-3x+1}{4x^2+2x+5} =$

2. $\lim_{x\to\infty} \frac{7x-2}{2x^2+x-11} =$

3. $\lim_{x\to-\infty} \frac{x^4-x+12}{x^2+2} =$

2. If $f(x) = \frac{3x^2+x}{3x^2-x}$ then $f'(x)$ is

a) 1
b) $\frac{6x^2+1}{6x^2-1}$
c) $\frac{-6}{(3x-1)^2}$
d) $\frac{-2x^2}{(x^2-x)^2}$
e) $\frac{36x^3-2x}{(x^2-x)^2}$

3. If $f(x) = 5x^{\frac{4}{3}}$ then $f'(8) =$

4. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-7x+12}{x-4}$ when $x \neq 4$, then $f(4) =$

6. If $f(x) = \sec x + \csc x$, then $f'(x) =$

a) 0
b) $\sec^2 x + \csc^2 x$
c) $\csc x - \sec x$
d) $\sec x \tan x + \csc x \cot x$
e) $\sec x \tan x - \csc x \cot x$

7. An equation of the line **normal** to the graph of $y = \sqrt{(3x^2 + 2x)}$ at $(2, 4)$ is

8. If $f(x) = \ln(\cos(3x))$, then $f'(x) =$

9. $\int x\sqrt{5x^2 - 4}\, dx =$

10. The slope of the line tangent to the graph of $3x^2 + 5\ln y = 12$ at $(2, 1)$ is:

11. The acceleration of a particle moving alone the x-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the particle is 4 when $t = 0$, then the particle is changing direction at

12. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

13. $\int (e^{3\ln x} + e^{3x})dx =$

a) $3 + e^{3x} + C$
b) $\frac{x^4}{4} + 3e^{3x} + C$
c) $\frac{e^{x^4}}{4} + 3e^{3x} + C$
d) $\frac{e^{x^4}}{4} + \frac{e^{3x}}{3} + C$
e) $\frac{x^4}{4} + \frac{e^{3x}}{3} + C$

14. If $f(x) = (x^2 + x + 11)\sqrt{(x^3 + 5x + 121)}$, then $f'(0) =$

15. If $f(x) = \cos^3(x+1)$, then $f'(\pi) =$

16. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$

17. $\frac{dy}{dx}\int_0^{3x} \cos(t)\,dt =$

18. $\int \tan^6 x \sec^2 x \, dx =$

19. Find a positive value c, for x, that satisfies the conclusion of the Mean Value Theorem for Derivatives (MVTD) for $f(x) = 3x^2 - 5x + 1$ on the interval $[2, 5]$.

a) 1
b) $\frac{13}{6}$
c) $\frac{11}{6}$
d) $\frac{23}{6}$
e) $\frac{7}{2}$

20. A particle's position is given by $s(t) = t^3 - 6t^2 + 9t$. What is the acceleration at time $t = 4$?

21. $\int \frac{\ln x}{3x} dx =$

22. Find the value of c that satisfies Rolle's Theorem for $f(x) = \frac{x^2+4x-12}{x^2+2x-3}$ on the interval $[-6, 2]$.

23. If $\cos^2 x + \sin^2 y = y$, then $\frac{dy}{dx} =$

24. If $f(x) = e^{3x}$, then $f''(\ln 3) =$

25. $\int 18x^2 \sec^2(3x^3)\, dx =$

26. $\int (x^2 + 2x)\cos(x^3 + 3x^2)\, dx =$

27. If $f(x) = x^{-3} + 3\sqrt{x} + 5\pi - e^2$, then $f'(x) =$

28. $\lim_{x \to 1}\left((x^3 + 2x^2 - 3)(x^{-2} + 7x)\right) =$

Calculator Possibly Required

22. $\int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{-\frac{\pi}{4}}^{0} \cos x\, dx =$

23. The graph of $y = 5x^4 - x^5$ has an inflection point (or points) at:

24. Graph the function. Approximate $\int_0^1 \sin^2 x\, dx$ using 4 rectangles (upper AND lower sums). Simplify both of your answers to three decimal places.

25. The average value of the function $f(x) = \ln^2 x$ on the interval $[2, 4]$ is
(hint: use your calculator to calculate the definite integral)

26. $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{6}+h\right)-\tan\left(\frac{\pi}{6}\right)}{h} =$

27. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at:

28. Boats A and B leave the same place at the same time. Boat A heads due north at 12 km/hr. Boat B heads due east at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?

29. A 20-foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor (in ft/sec)?

30. Consider the equation $x^2 - 2xy + 4y^2 = 64$.
    a. Write an equation for the slope of the curve at any point (x, y).
    b. Find the equation of the tangent lines to the curve at the point $x = 2$.
    c. Find $\frac{d^2y}{dx^2}$ at $(0, 4)$.

31. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.
    a. Find an equation of the line tangent to the graph at $(-2, 17)$.
    b. Find the x- and y-coordinates of the relative maxima and relative minima. Verify your answer.
    c. Find the x- and y-coordinates of the points of inflection. Verify your answer.